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NEW APPROACHES TO LINEAR AND NONLINEAR PROGRAMMING

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Subj: RETURNED GRANTEE/CONTRACTOR TECHNICAL REPORTS

1. This confirms our conversations of 27 Feb 97 and 11 Jul 97. Enclosed are a number of technical reports which were returned to our agency for lack of clear distribution availability statement. This confirms that all reports are unclassified and are "APPROVED FOR PUBLIC RELEASE" with no restrictions.

2. Please contact me if you require additional information. My e-mail is *silverr@onr.navy.mil* and my phone is (206) 625-3196.

ROBERT J. SILVERMAN

SUMMARY OF COMPLETED PROJECT

The project explored the theoretical properties and computational performance of algorithms for solving constrained optimization problems (linear and nonlinear programs). Particular emphasis was placed on algorithms for solving *large* problems.

The practical applications of optimization are innumerable. For example, mathematical models of the economy (to analyze the optimal use of natural resources) are typically large linear or nonlinear programs. Areas in which we have been actively involved include optimal generation and transmission of electricity, optimization of aircraft and spacecraft trajectories, optimal structural design, and financial modeling such as portfolio optimization.

Progress on solution algorithms and software for such applications is ultimately reflected in improved techniques in many other areas of science and industry.

TECHNICAL INFORMATION

The research described was conducted by the Principal Investigators and their students (Meredith Goldsmith, Erik Boman, Sam Eldersveld, Dulce Ponceleón) whose were work was supported by the grant. The main achievements during the grant period are listed below in various categories.

SQP Methods

In Sequential Quadratic Programming (SQP) methods for constrained optimization, most of the computational effort goes into solving QP subproblems to generate each search direction.

- Theoretical and practical results were obtained for SQP algorithms based on incomplete solution of the QP subproblems [MP95].
- A large-scale SQP algorithm and its implementation LSSQP [Eld92] demonstrated the feasibility of working with *reduced Hessians* and *sparse Jacobians*.
- A new large-scale SQP algorithm and its implementation SNOPT [GMS93, GMS96b] underwent continuous development. The application of this new software to the important class of trajectory optimization problems has invoked considerable interest in the U.S. and European aerospace communities. It is compatible with our existing general-purpose optimizer MINOS [MS95], and will be more efficient for problems whose nonlinear functions and gradients are expensive to evaluate. It also has a more reliable mechanism for ensuring convergence on problems with nonlinear constraints.

Interior-Point Methods

- Primal-dual interior-point algorithms for linear programming were developed and implemented [GMPS94, GMPS95].
- A primal interior-point algorithm for convex programming was developed and implemented in prototype (MATLAB) form [JS95].

- Special linesearch procedures were developed for use with logarithmic barrier functions [MW94].

- A series of papers [GSS96, Sau95, Sau96] explored the stability of LDL^T factorizations for solving large *indefinite* systems of equations—notably, the augmented systems arising in sparse least-squares problems and in barrier methods for linear and quadratic programming [GMPS94].

Other Optimization Methods

- A new second-derivative method for unconstrained optimization was formulated [FGM95]. The method is based on finding a partial Cholesky factorization of the Hessian. The new method is one of the most efficient available for practical computation.

- A Newton approach to large-scale optimization with linear inequality constraints was developed in [FM94].

- An intriguing new simplex-type method for linear programming was developed in [CPS93]. This bears resemblance to Lemke's method, but is at least four times as efficient in terms of work per iteration.

- A new dense quadratic programming code QPOPT was completed and documented [GMS95b].

- The large-scale optimizer MINOS underwent continual development, including the implementation of a callable subroutine [MS95].

Publications

The following papers were referred to above. They have been published by the Principal Investigators during the review period.

[CPS93] H. Chen, P. M. Pardalos and M. A. Saunders (1993). The simplex algorithm with a new primal and dual pivot rule, *Operations Research Letters* 16, 121–127, 1994.

[CM95] J. W. Chinneck and M. A. Saunders (1995). MINOS(IIS) version 4.2: Analyzing infeasibilities in linear programming, *European J. of Operational Research* 81, 217–218.

[Eld92] S. K. Eldersveld (1992). *Large-scale Sequential Quadratic Programming*, PhD thesis, Report SOL 92-4, Dept of Operations Research, Stanford University.

[FGM95] A. Forsgren, P. E. Gill and W. Murray (1995). Computing modified Newton directions using a partial Cholesky factorization, *SIAM J. on Matrix Analysis and Applications* 16, 1, 139–150, 1995.

[FM94] A. Forsgren and W. Murray (1994). Newton methods for large-scale linear inequality-constrained minimization, Report SOL 94-3, Dept of Operations Research, Stanford University.

[GMS93] P. E. Gill, W. Murray and M. A. Saunders (1993). Large-scale SQP methods and their application in trajectory optimization, in R. Bulirsch and D. Kraft (eds.), *Control Applications of Optimization*, International Series of Numerical Mathematics, Birkhauser, Basel, Boston, Stuttgart.

[GMPS94] P. E. Gill, W. Murray, D. B. Ponceleón and M. A. Saunders (1994). Solving reduced KKT systems in barrier methods for linear programming, in G. A. Watson and D. Griffiths (eds.), *Numerical Analysis 1993*, Pitman Research Notes in Mathematics 303, Longmans Press, 89–104.

[GMPS95] P. E. Gill, W. Murray, D. B. Ponceleón and M. A. Saunders (1995). Primal-dual methods for linear programming, *Mathematical Programming* 70, 251–277.

[GMS95a] P. E. Gill, W. Murray and M. A. Saunders (1995a). Fortran software for optimization, Proceedings of the NSF Design, Manufacturing and Industrial Innovation Grantees Meeting, University of California, San Diego, CA (January 4–6, 1995).

[GMS95b] P. E. Gill, W. Murray and M. A. Saunders (1995b). User's guide for QPOPT 1.0: A Fortran package for quadratic programming, Report SOL 95-4, Dept of Operations Research, Stanford University.

[GMS96a] P. E. Gill, W. Murray and M. A. Saunders (1996a). SQP methods for large-scale optimization, Proceedings of the NSF Design, Manufacturing and Industrial Innovation Grantees Meeting, University of New Mexico, Albuquerque, NM (January 3–5, 1996).

[GMS96b] P. E. Gill, W. Murray and M. A. Saunders (1996b). SNOPT: An SQP algorithm for large-scale constrained optimization, Report SOL 96-xx, Dept of Operations Research, Stanford University (to appear).

[GSS96] P. E. Gill, M. A. Saunders and J. R. Shinnerl (1996). On the stability of Cholesky factorization for quasi-definite systems, *SIAM J. on Matrix Analysis and Applications* 17(1), 35–46.

[JS95] F. Jarre and M. A. Saunders (1995). A practical interior-point method for convex programming, *SIAM J. on Optimization* 5, 149–171.

[MP95] W. Murray and F. J. Prieto (1995). A sequential quadratic programming algorithm using an incomplete solution of the subproblem, *SIAM J. on Optimization* 5, 589–639.

[MW94] W. Murray and M. H. Wright (1994). Line search procedures for the logarithmic barrier function, *SIAM J. on Optimization* 4, 2, 229–246.

[MS95] B. A. Murtagh and M. A. Saunders (1995). MINOS 5.4 User's Guide, Report SOL 83-20R, Dept of Operations Research, Stanford University, revised January 1987, March 1993, February 1995.

[Sau95] M. A. Saunders (1995). Solution of sparse rectangular systems using LSQR and CRAIG, *Nordisk Tidskr. Informationsbehandling (BIT)* 35 , 588–604.

[Sau96] M. A. Saunders (1996). Cholesky-based methods for sparse least squares: The benefits of regularization, in L. Adams and J. L. Nazareth (eds.), *Linear and Nonlinear Conjugate Gradient-Related Methods*, Proceedings of AMS-IMS-SIAM Joint Summer Research Conference, University of Washington, Seattle, WA (July 9–13, 1995), SIAM, Philadelphia, 92–100.